Those who seek real contemplation must therefore abandon the intellect. Intellect will never give them wholly disinterested knowledge. This is why Bergson tells us to give up trying to get knowledge of reality by way of conceptual thought, and make instead the great effort which pure intuition demands, not because he despises pure speculation but just because he prizes it so highly.

If the reality which pure intuition reveals should turn out in the end to be mere "activity without purpose" not "inspired by some vision, some imaginative foreshadowing of a world less painful, less unjust, less full of strife than the world of our every-day life ....," then, doubtless, only those who care more for truth than for a pleasing picture will welcome the new philosophy.

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"MULTIPLICATION OF PEARS AND PENCE."

Under the caption, "Multiplication of Pears and Pence," a letter by Mr. Frederic Hovenden is discussed in the October *Monist*, and it is suggested that "some one who believes in algebra" may be found who is willing to defend the use of concrete operators or of concrete numbers in equations.

The fact is that not only are concrete operators and complex units perfectly logical, but in each of the three lines in which arithmetic and algebra are actually used men have come to use concrete operators incessantly, and they *think in* complex units. These three lines are engineering, science and business. Feet per second, ton-miles, cents per yard, watthours, acre-feet, car-miles, pounds per square inch, and a host of other complex units have long ago become single ideas.

Not only physicists and engineers but every one thinks in complex units. 60 mi./hr. is not 60 miles nor 60 hours, but it is 60 *units of velocity,* the unit being a *mile-per-hour.* A railroad hauling 50 tons 200 miles gets paid, not for 50 tons nor yet for 200 miles, but for

$$50 \text{ tons} \times 200 \text{ miles} = 10{,}000 \text{ ton-miles.}$$

The area of a rectangle 8×15 centimeters is

$$8 \text{ cm.} \times 15 \text{ cm.} = 120 \text{ sq. cm.}$$

If we buy 20 pears for 30 d. the cost of pears is

$$30 \text{ d.} \div 20 \text{ pears} = 1.5 \text{ d./pear.}$$

And again note that the cost of pears is not 30, nor 30 d., nor 20, nor 20 pears, nor 1.5, nor 1.5 d., but *it is* 1.5 *pence-per-pear.*

Says Professor John Perry, "If I were asked to multiply 2 tables by 3 chairs I would not refuse. I would say 6 chair-tables. But if I were asked to say what I mean by a chair-table I would refuse, because nobody has ever given a meaning to the term. But I do know that when this sort of thing comes into a Physical Problem we can always give a useful meaning."[1]

One may also point to the "method of dimensions" so much used in checking formulas, etc. in physics and engineering, by writing out the "dimensions" of the quantities on each side of the equation in terms of length, mass, and time, or the like, to see whether the number of times these quantities appear, is the same on the two sides of the equation. One of the most remarkable points in the history of science is the rôle which dimensions have played in the development of electromagnetic theory. Electric and magnetic quantities may be measured in either of two systems, yet the "dimensional formulas" are not the same in the two systems but have the ratio (length $\div$ time), which is a velocity. In Maxwell's electro-magnetic theory this velocity is the velocity of light. Experiment has shown this to be true, so that the study of dimensions in equations, that is of concrete multipliers and divisors, has played a most important part in the development of the theory.

To one who has not followed the powerful methods of vector analysis as applied to physics, it is really surprising to learn the extent to which this matter of complex units is carried. Not only does a single symbol represent a very complex unit, but its direction in space is also included in it. Yet the whole complex unit, together with the direction cosines determining its direction in space, becomes a single idea.

From a philosophical standpoint there can be no objection to a complex unit, or to concrete operators, and to one who is accustomed to take a pragmatic attitude of mind, it is decidedly desirable to use them since clearness is surely added by doing so.

Great harm has been done the cause of education by insisting that operators should be abstract.[2] The demand of the pure mathematicians who have written so many of the elementary text-books,

[1] John Perry, *Practical Mathematics,* London, 1899.

[2] For a discussion of the matter from a pedagogical standpoint, see a paper by P. G. Agnew, "Should Concrete Multipliers and Divisors be Allowed?" *Popular Educator,* 28, p. 229, January, 1811.

and the insistence on the part of teachers on set forms of analysis have brought this about. The following example is probably a fair average of the methods actually in use in the schools:

"If $5\frac{1}{2}$ yards cost 99 cents, the *number* of cents per yard which the cloth costs is the *number* of times $5\frac{1}{2}$ is contained in 99, or 18."

Who can say that such a benumbing circumlocution makes the matter clearer to the child's mind, (or to an adult mind other than that of a pure mathematician), than merely to say

$$99 \text{ cents} \div 5\tfrac{1}{2} \text{ yards} = 18 \text{ cents per yard?}$$

The latter presents the whole process of reasoning, both logic and details in a single step.

P. G. AGNEW.